Detection of Whether The Autocorrelated Meteorological Time Series Have Stationarity by Using Unit Root Approach: The Case of Tokat

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Abstract: This paper presents methodologies on detecting nonstationarity and removing nonstationarity by differencing for the autocorrelated time series. For this purpose, 5-year daily temperature, light intensity and relative humidity data from weather station in Tokat were used as materials. Since 5-year daily data sequences include many records, the new data sequences were constituted by averaging daily average temperature and relative humidity (average of records at 7.00, 14.00 and 21.00 in a day) and daily light intensity of five years. The existence of serial correlation between the averages from the mentioned climatic components was examined by using graphical approach, Ljung-Box Q and Runs tests. These three approaches imply that the averages have serial correlation. Unit root test (augmented Dickey and Fuller, ADF) was applied to test whether the averaged daily data sequences are nonstationary. The results of ADF emphasis the existence of nonstationarity in the daily data sequences. Similarly, the autocorrelation function graphs (correlogram) show nonstationarity, because of slowly decay in the autocorrelation functions for the daily data sequences. The first differencing was applied to remove nonstationary. After taking the first differencing, the ADF test results and the correlograms of the daily data sequences showed stationary.

Key Words: Unit root, Dickey and Fuller test, Ljung-Box Q statistic, runs test.

1. Introduction

The series have been referred as the autocorrelated time series if there is dependence between observations of a given series. Especially, there is a significant dependence in observations recording of many hydrologic phenomena. As known, observations of daily discharges do not change appreciably from one day to another. There is a tendency for the values to cluster, in the sense that high values tend to follow high values and low values tend to follow low values. Thus, the daily discharges are not independently distributed in time. The dependence among monthly discharges is less than that among daily discharges, and the dependence among annual discharges is less than that among monthly discharges. Thus, the dependence between hydrologic observations decreases with an increase in the time base. If there is a linear dependence between the values of a series, the correlation between the values may be taken as dependence criterion (Chow, 1964).

Many hydrologic time series processes may be stationary or nonstationary. Nonstationary time series can occur in many different ways. They could have nonconstant
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means, time varying second moments such as nonconstant variance, or have both of these properties (Wei, 1990).

The occurrence of nonstationarity in a hydrologic time series can result from gradual natural or man-induced changes in the hydrologic environment producing the time series. Changes in watershed conditions over period of several years can result in corresponding changes in streamflow characteristic that show up as trends in time series of streamflow data. Urbanization on a large scale may result in changes in precipitation amounts that show up as trends in precipitation. (Huff and Changnon, 1973). Besides, natural events such as earthquakes, large forest fires and landslide that quickly and significantly alter hydrologic regime of an area cause jumps in the time series. Also, jumps in the time series results from man-made changes such as a new dam construction, and the beginning or cessation of pumping of ground water (Bayazit, 1981).

Especially, in stochastic modeling studies, nonstationarity is a fundamental problem. Therefore, a time series, which has nonstationarity, should be converted to a stationary time series. A nonstationary time series may be transformed to a stationary time series by using linear difference equation. But, it is needed to detect whether a given series is nonstationary before transforming. For this reason, there are alternative approaches as graphical method, nonparametric tests and unit root test. In addition to than, the autocorrelation function (ACF) is very important tool for the autocorrelated series. Enders (1995) expresses that inspection of ACF serves as a rough indicator of whether nonstationarity is present in a series. Wei (1990) states that if the sample ACF decays very slowly, it indicates that differencing is needed. This inspection of ACF implies that the sample is nonstationary.

In time series analysis, the most commonly used transformations are variance-stabilizing transformations and differencing. Since differencing may create some negative values, variance-stabilizing transformations should be applied before taking differencing (Cromwell et al., 1994)

Wilson (1990) expresses that the hydrology of a region depends primarily on climate of the region and, climate is significantly affected by the geographical position on the earth’s surface. Therefore, having information about the conditions related to the hydrology of a region in future is very important for hydrologists. For this reason, especially, modeling studies on climatic components are rather needed. The present study is an attempt to detect whether time series data related to temperature, light intensity and relative humidity as known climatic components are nonstationary and, to remove nonstationarity by differencing.

2. Material and Method

2.1 Study Area

Tokat province selected as study area is located between 39º 45' N and 40º 45' N latitudes, 35º 30' E and 37º 45' E longitudes, covering approximately 10160.7 km². About 30% of the area is occupied by cropland. Wheat is the major food crop (average sowing area is 68.5% of the total cropped area) not only in the district, but in the entire Turkey. The major sources of irrigation are rainfall, canal and groundwater.

In this study, 5-year daily temperature, light intensity and relative humidity data from weather station in Tokat-TURKEY were used as materials. Since 5-year daily data sequences include many records, the new data sequences were constituted by averaging daily average temperature and relative humidity (average of records at 7.00, 14.00 and 21.00 in a day) and daily light intensity of five years.

2.2 Testing for Nonstationarity

Nonstationarity is the first fundamental statistical property tested in time series analysis. A nonstationary time series has no long-run mean and its variance is time-dependent. If nonstationarity is present in a given time series, it is possible to transform the series to a stationary series. Because of most time series data are nonstationary, transformation is needed in stochastically modeling. In this sense, the most common transformation is differencing, that is, subtracting a past value of a variable from its current value (Greene, 2000). But, it is necessary to detect whether nonstationarity is present in a series before differencing.

To detect whether a given series has nonstationarity, lets assume that the relationship between current value (in time t) and last value
(in time $t-1$) in the time series is as following (Enders, 1995):

$$y_t = a_1 y_{t-1} + \varepsilon_t$$  \hfill (1)

Where, $\varepsilon_t$ is a white noise process. This model is a first order autoregressive process. In the process, if $|a_1| < 1$, the series is referred as stationary. The $y_t$ series is nonstationary if $|a_1| = 1$ or $>1$. The series has a unit root and such processes are called as random walk. Having a unit root means that the effect of past shocks continues.

Greene (2000) stated that a nonstationary time series could be converted to a stationary time series by taking first or higher order difference. If $y_{t-1}$ is subtracted from the right and left sides of the above equation, the new equation is yielded as following:

$$\Delta y_t = (a_1 - 1)y_{t-1} + \varepsilon_t$$  \hfill (2)

This equation is expressed as a first order difference equation. If $a_1$ is taken one (1) in the equation, the effect of unit root can be removed from the actual series that has nonstationarity via a first differencing. The series that is stationary with the first difference is said to be integrated of order one and, is denoted by I (1). If the series becomes stationary after being differenced d times, the nonstationary series is integrated of order d, and is denoted by I (d). Although there are different approaches to test unit root, Dickey and Fuller test is the most popular.

2.3 Dickey and Fuller Test

One of unit root approaches commonly used to explain whether a time series is nonstationary is Dickey and Fuller test by Dickey and Fuller (1981). There are three types of equation to test for unit root in Dickey and Fuller, which are:

A pure random walk, (if $a_1$ is taken, instead of $a_1-1$ in Equation 2)

$$\Delta y_t = a_1 y_{t-1} + \varepsilon_t$$  \hfill (3)

A random walk with drift (constant),

$$\Delta y_t = \alpha_0 + a_1 y_{t-1} + \varepsilon_t$$  \hfill (4)

A random walk with drift and linear time trend,

$$\Delta y_t = \alpha_0 + a_1 y_{t-1} + \beta t + \varepsilon_t$$  \hfill (5)

Where $\alpha_0$ and $\beta$ are the coefficients of constant and time trend, respectively. The null hypothesis related to the above equations is $H_0$: $y(t)$ is random walk, if $a_1 = 0$, because of $|a_1| = 1$. The associated alternative hypothesis for null hypothesis of each equation is $H_1$: $a_1 \neq 0$.

For each case, the test statistic related to the null hypothesis is calculated as (Cromwell et al., 1994):

$$[\tau_{cal} = \left(\frac{(\alpha_1 - 0)/SE(\alpha_1)}{}}\right)]$$  \hfill (6)

Where SE ($\alpha_1$) refers to standard error of $\alpha_1$. The calculated statistic, ($\tau_{cal}$), is compared with $\tau$-critical value from McKinnon (1990) at the chosen significance level and, if $\tau_{cal}$ is greater than $\tau$-critical value, the null hypothesis is rejected.

Enders (1995) stated that residuals were assumed to be independent and to have a constant variance in Dickey and Fuller test. Under the conditions that residuals have serial correlation, $\sum_{j=1}^{p-1} \Delta y_{t-j}$ term should be augmented to remove serial correlation in residuals to Dickey and Fuller test regressions (3, 4 and 5). This approach is called as augmented Dickey and Fuller Test. Where p is the number of lags chosen for dependent variable (residuals). The test variations related to the augmented Dickey and Fuller regressions are as given above. Gujarati (2002) suggested that the existence of autocorrelation in the residuals could be achieved by Durbin-Watson (DW) test.

Besides, the unit root approach in detecting nonstationarity, it is possible to visually determine via the autocorrelation function (ACF) from an autocorrelated series. Wei (1990) expresses that a slowly decaying ACF is indicative of a large characteristic root. This indicates that differencing is needed to convert series to stationary series.
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2.4 Inspection of Autocorrelation Function

Autocorrelation refers to the correlation of a time series with its own past and future values. Autocorrelation is called as serial correlation. Many hydrologic time series exhibit significant serial correlation. That is, the value of random variable under consideration at one period is correlated with the values of the random variable at earlier time periods. Positively autocorrelated series represent that high values tend to follow high values and low values tend to follow low values. Negatively autocorrelated series is characterized by reversals from high to low or from low to high (Box and Jenkins, 1976). The $k^{th}$ order autocorrelation coefficient of a given series is denoted as $r_k$ and defined as:

$$r_k = \frac{\sum_{t=k+1}^{n} x_t x_{t-k}}{\sum_{t=1}^{n} x_t^2}$$  \hspace{1cm} (7)

Hipel et al. (1977) suggest that the ACF for a series should not exceed a maximum lag of approximately $n/4$. If the autocorrelation function (ACF) of a given time series, $x(t)$, is significantly different from zero, this implies that there is dependence between observations. Therefore, ACF is a powerful complementary beneficial tool for testing independence (Ferguson et al., 2000). For this reason, the correlogram is drawn by plotting $r_k$ against lag $k$. If the autocorrelation coefficients from a given series fall in the confidence interval at 5%, the series is referred as independent. Controversially, if more than 5% of the serial correlation coefficients fall outside the limits, the series is called as dependent. This emphasis that the observations are correlated (Janacek and Swift, 1993). Salas et al. (1980) suggest that the confidence limits on the correlogram of a series can be calculated by the following equation.

$$CL = \frac{-1 \pm 1.96 \sqrt{n-k-1}}{n-k}$$  \hspace{1cm} (8)

Additional to graphical approach to render whether a series is autocorrelated, Ljung-Box Q statistic and runs tests were used as alternative approaches in this study. The approaches are given as following, respectively.

2.4.1 Ljung-Box Q (LBQ) Statistic

The Ljung-Box Q or $Q(r)$ statistic (Ljung and Box, 1978) can be employed to check independence instead of visual inspection of the sample autocorrelations. A test of this hypothesis can be done for serial dependence by choosing a level of significance and then comparing the value of calculated $\chi^2$ with $\chi^2$-table of critical value. If the calculated $\chi^2$ value is smaller than the $\chi^2$-table critical value, there is no serial dependence on the basis of available data. The $Q(r)$ statistic is calculated by using:

$$Q(r) = n(n+2)\sum_{k=1}^{m} (n-k)^{-1} r_k^2$$  \hspace{1cm} (9)

Where $m$ is the number of lags.

2.4.2 Runs Test

The runs test can be used to decide if a data set is from a random process. A run is defined as a series of increasing values or a series of decreasing values. The number of increasing, or decreasing, values is the length of the run. In a random data set, the probability that the $(I+1)^{th}$ value is larger or smaller than the $I^{th}$ value follows a binomial distribution which forms the basis of the runs test. The first step in the runs test is to compute the sequential differences ($Y_i - Y_{i-1}$). Positive values indicate an increasing value whereas negative values indicate a decreasing value. In other term, if $Y_i > Y_{i-1}$, a 1 (one) is assigned for an observation and a 0 (zero) otherwise. The series then has an associated series of 1’s and 0’s. A run is a consecutive sequence of 0’s or 1’s. A run’s test check if the number of runs is the correct number for a series that is random. To figure this out, let $T$ be the number of observations, $T_A$ be the number above the mean and $T_B$ the number below the mean. Let $R$ be the observed number of runs. Then using combinatorial methods, the probability $P(R)$ can be established and mean and variance of $R$ can be derived: (Gibbons, 1997; Cromwell et al., 1994). When $T$ is relatively large ($>20$), the distribution of $R$ is approximately normal.
The null hypothesis is rejected if the calculated $Z_N$ value is greater than the selected critical value obtained from standard normal distribution table. In other words, $x(t)$ series is decided to be non-random.

3. Results and Discussion

The autocorrelation function (ACF) graphs, which is known as correlogram, of the data were taken into consideration to visually detect the existence of nonstationarity in the averaged daily temperature, light intensity and relative humidity data from 5-year daily records at weather station in Tokat-TURKEY (Figure 1). The figure indicates that ACFs decline gradually implying nonstationary. Especially, slowly decay in ACFs related to the averaged daily temperature and light intensity data sequences is more obvious than the ACF of the averaged daily relative humidity data sequences. Also, the ACFs show that the averaged daily data sequences have rather high serial correlation. Similarly, Ljung-Box Q (LBQ) and Runs test results emphasize rather high serial correlation between the averages of the daily data sequences (Table 1). The LBQ statistic calculated for each of three daily data sequences were rather high compared with $\chi^2$ critical values at 5% significance level. Because the LBQ statistics are greater than the $\chi^2$ critical values, the null hypothesis of independence between the averages is rejected. Addition to LBQ statistic, runs test results show serial correlation between the averages. $Z_N$ values obtained for each data sequences are greater than $Z$ critical value from standard normal distribution table at 5% significance level. This implies rejection of the null hypothesis related to no serial correlation. Because the ACFs attenuate very slowly, nonstationarity in the daily data sequences should be counteracted for performing stationary data sequences. Therefore, the most common transformation, differencing, was applied to the averaged daily data sequences. Figure 2 illustrates that the first differencing removes nonstationary pattern for each daily data sequences.

Unit root test (augmented Dickey and Fuller, ADF,) was applied to the averaged daily data sequences to test the nonstationary of data sequences. Test results were given in Table 2. The ADF test statistics, ($t_{ad}$), for the daily data sequences were smaller than the critical values obtained from MacKinnon (1990) at 0.01, 0.05 and 0.10 significance levels, except 10% significance level for the averaged daily humidity data. According to these results, the null hypothesis, which has a unit root, for the data sequences should be accepted at 0.01, 0.05 and 0.10 significance levels, except daily relative humidity data at 10% significance level. For each of the averaged daily data sequences from 5-year daily temperature, light intensity and relative humidity data, maximum lag lengths for ADF test were selected as 6, 9 and 5, respectively. Maximum lag lengths of ADF test were selected by taking into consideration Akaike Information Criterion (AIC) given by Wei (1990). The model in which the AIC is the lowest is chosen for maximum lag length. AIC for maximum lag lengths (6, 9 and 5, respectively) related to temperature, light intensity and relative humidity data are calculated, respectively, as 0.847, 4.732 and 1.546. Whether the existence of autocorrelation in the residuals from the regression models with the lowest AIC obtained for the daily data sequences was fulfilled by Durbin-Watson (DW) test. The DW test values related to temperature, light intensity and relative humidity data are 2.004, 2.012 and 1.995, respectively. Gujarati (2002) expresses that if DW statistic is 2, there is no serial correlation in the residuals. There is no serial correlation between the residuals, because the DW statistics for the residuals from the regression models with the lowest AIC are very close to 2. The values (V) of the parameter associated the standard errors (SEV), $t$-ratios ($t_{ad}$) for drift (constant) and trend parameters in regressions of the daily data sequences were given in Table 2. These $t$-ratios ($t_{ad}$) related to constant and trend coefficients were compared with critical value of 1.96 obtained from $t$-
distribution at 0.05 significance level. Only, t-ratios \( t_{cal} \), of constant and trend coefficients related to light intensity data and constant coefficient of relative humidity data were greater than critical value (Table 2). Therefore, these parameters concerning with light intensity and relative humidity data should be involved in its regressions.

### Table 1. Test Results of Serial Correlation from Actual Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Ljung-Box Q Test</th>
<th>Runs Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Q(r) )</td>
<td>( \chi^2 )</td>
</tr>
<tr>
<td>Temp.</td>
<td>11037.2</td>
<td>112.02</td>
</tr>
<tr>
<td>Light Int.</td>
<td>9513.4</td>
<td>NR</td>
</tr>
<tr>
<td>Rel.Hum.</td>
<td>2406.4</td>
<td>NR</td>
</tr>
</tbody>
</table>

NR, the variable are dependent
Temp., Temperature
Light Int., Light Intensity
Rel.Hum., Relative Humidity

### Table 2. Unit Root Test Results for The Actual Data Sequences

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF Statistic</th>
<th>Test Critical Value</th>
<th>Constant</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \delta )</td>
<td>( \delta ) 0.01</td>
<td>( \delta ) 0.05</td>
<td>( \delta ) 0.10</td>
</tr>
<tr>
<td>Temp.</td>
<td>-0.735</td>
<td>1.922</td>
<td>-3.135</td>
<td>-0.003</td>
</tr>
<tr>
<td>Light Int.</td>
<td>-1.095</td>
<td>-0.577</td>
<td>-3.424</td>
<td>-0.005</td>
</tr>
<tr>
<td>Rel.Hum.</td>
<td>-3.215</td>
<td>-0.577</td>
<td>-3.424</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

ADF test was applied to detect whether the differenced series are stationary after performing the first differencing to the averaged daily data sequences. Test results were given in Table 3. The ADF test statistics of the daily data sequences were greater than the MacKinnon critical values at 0.01, 0.05 and 0.10 significance levels. According to these results, the null hypothesis, which has a unit root, should be rejected at 0.01, 0.05 and 0.10 significance levels. For each of the averaged daily data sequences, maximum lag lengths of ADF test were 5, 8 and 4, respectively. The lowest AIC for maximum lag lengths (5, 8 and 4, respectively) related to the differenced temperature, light intensity and relative humidity data were calculated, respectively, as 0.843, 4.730 and 1.569. DW test values concerning with whether serial correlation in the residuals from the regression models selected for the differenced data sequences is present, were calculated, respectively, as 2.005, 2.013 and 2.002. The values (V) of the parameter associated the standard errors (SEV), t-ratios \( t_{cal} \) for drift (constant) and trend parameters in regressions for the differenced data sequences were given in Table 3. These t-ratios \( t_{cal} \) related to constant and trend coefficients of the differenced temperature and light intensity data (except constant parameter of temperature data) were greater than critical value of 1.96 obtained from t-distribution at 0.05 significance level. Therefore, these parameters concerning with the differenced temperature and light intensity data should be present in its regressions.

### Table 3. Unit Root Test Results for The Differenced Data Sequences

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF Statistic</th>
<th>Test Critical Value</th>
<th>Constant</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \delta )</td>
<td>( \delta ) 0.01</td>
<td>( \delta ) 0.05</td>
<td>( \delta ) 0.10</td>
</tr>
<tr>
<td>Temp.</td>
<td>-11.034</td>
<td>-3.135</td>
<td>-3.424</td>
<td>-0.005</td>
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<tr>
<td>Light Int.</td>
<td>-9.968</td>
<td>-3.424</td>
<td>-3.135</td>
<td>-0.005</td>
</tr>
<tr>
<td>Rel.Hum.</td>
<td>-13.501</td>
<td>-0.577</td>
<td>-0.577</td>
<td>0.0001</td>
</tr>
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</table>
Figure 1. ACF-The Actual Data
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Figure 2. ACF-The Differenced Data
4. Conclusion

One of the fundamental problems in stochastic modeling is nonstationarity of a given time series. Therefore, nonstationary time series should be transformed to a stationary time series. There are different ways to detect whether a given series is nonstationary. Unit root test (Augmented Dickey and Fuller test) is one of the most commonly used approaches.

In this study, Augmented Dickey and Fuller (ADF) test was applied to 5-year daily temperature, light intensity and relative humidity data from the weather station in Tokat province. The ADF test showed that the daily data sequences are nonstationary. To remove nonstationarity in the daily data sequences, first differencing was applied to the daily data sequences. ADF test was applied to detect whether the differenced series are stationary after the first differencing. The ADF test results showed that the first differencing helped to remove nonstationary from the data.

References